AC part (02) [1]

[1] Capacitors

symbol —||— two plates charged with ac Source

$\rightarrow$ Capacitance $= C = \dfrac{Q \rightarrow \text{charge}}{V \rightarrow \text{applied voltage}} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

$\varepsilon_0 \rightarrow 8.85\times10^{-12}$ F/m

$\varepsilon_r \rightarrow$ ثابت العازلية permitivity

$A \rightarrow$ area of plates

$d \rightarrow$ dist. betw plats

or $Q = CV$

$\rightarrow$ its Unit is Farad (F) $\rightarrow$ actually (micro, nano, pico farad)

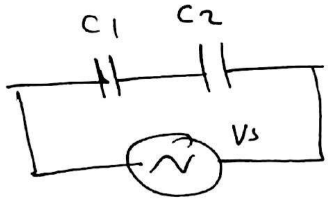
EX(1) A certain capacitor stores 50 microcolombs with 10V across its plats. What is capacitance (in microfarad)

Sol/ $C = Q/V = \dfrac{50 \text{ micro}}{10} = 5\mu F$

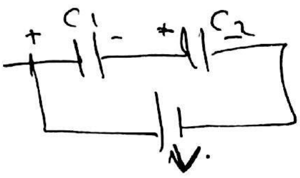
EX(2) determine the capacitance of parallel plats having area $0.01 m^2$ & separation 0.5 mil $= [1.27\times10^{-5} m]$, Dielectric is mica (has ε_r of 5)

Sol $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d} = \dfrac{(8.85\times10^{-12})(5)(0.01)}{1.27\times10^{-5}} = 0.035 \mu f$

a- Series Connection of Capacitors



or



note if $C_1 = C_2 = C_3 = \dots$

$\therefore C_T = \dfrac{C}{n}$ $n \rightarrow$ عدد المكثفات

$V_T = V_1 + V_2 + \dots$

$Q_1 = Q_2 = Q_3 \dots$

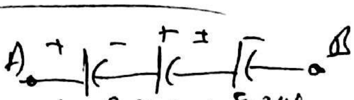
$\dfrac{Q_T}{C_T} = \dfrac{Q_1}{C_1} + \dfrac{Q_2}{C_2} + \dots$

$\therefore \boxed{\dfrac{1}{C_T} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots}$

الخلاصة المكثفات series =
تتعامل معاملة مقاومات = parallel

∴ Series Capacitor Like Parallel Resistors

EX(3) Find total capacitance between A & B



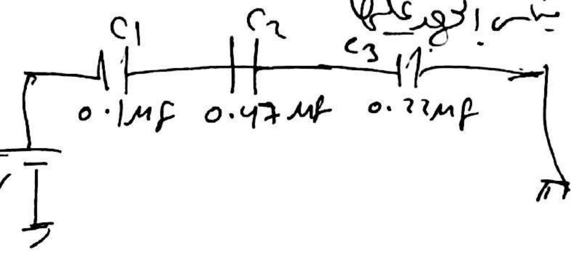
$\dfrac{1}{C_T} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$ $\therefore \dfrac{1}{C_T} = \dfrac{1}{2.3\mu f}$ $\therefore C_T = 2.3\mu f$

note if you want to use voltage divider in series connection to determine voltage across each individual capacitor, So:-

أي $V_1 = V_T \times \frac{X_{C1}}{X_{CT}} = V_T \cdot \frac{C_T}{C_1}$ → total cap

قيمة المتسعة المراد
ايجاد الجهد عليها

EX(4) find voltage across each capacitor



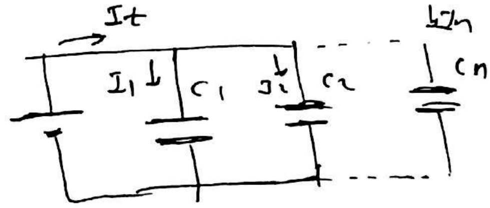
$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = 0.06\mu f$

$V_1 = V_T \frac{C_T}{C_1} = 25 \times \frac{0.06\mu}{0.1\mu} = 15V$

$V_2 = V_T \frac{C_T}{C_2} = 25 \times \frac{0.06\mu}{0.47\mu} = 3.19V$

$V_3 = V_T \frac{C_T}{C_3} = 25 \times \frac{0.06\mu}{0.22\mu} = 6.81V$

b- Parallel Connection of C



$Q_T = Q_1 + Q_2 + Q_3 + \cdots Q_n$

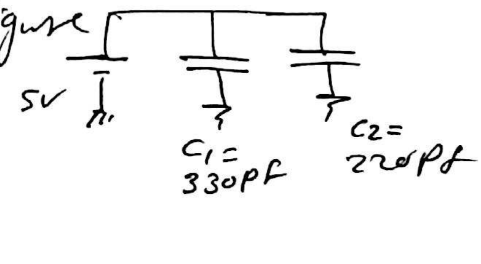
$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3 + \cdots + C_n V_n$

but $V_T = V_1 = V_2 = \cdots V_n$

$\therefore \boxed{C_T = C_1 + C_2 + C_3 + \cdots + C_n}$

So parallel C ≡ series R

EX(5) what is the voltage across each cap of figure

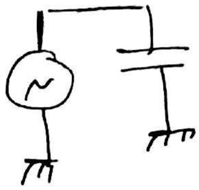


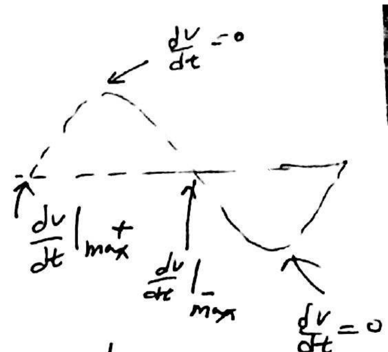
$C_T = C_1 + C_2 = 330 + 220 = 550pf$

$V_T = V_1 = V_2 = 5V$

in Capacitor

$$i = C\frac{dv}{dt}$$

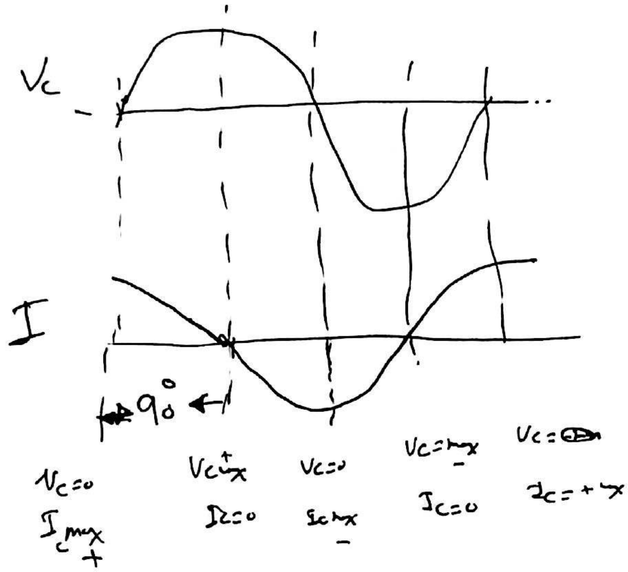


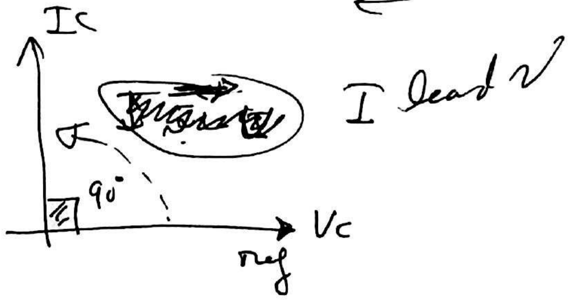


Relation between I & V in capacitor

~~V~~ I lead ~~I~~ V by 90° in pure capacitive circuit

يسبق v





I lead v

Capacitive Reactance

المقاومة السعوية للمكثف

$$X_C = \frac{1}{2\pi f C}$$

magnitude

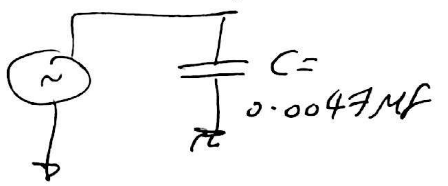
, also written $-jX_c$ or $\frac{1}{jX_c}$

phase (90°)

EX(6) For the circuit shown, Determine Capacitive Reactance if f = 1KHz

Sol:

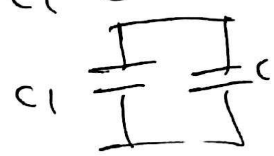
$$X_C = \frac{1}{2\pi f c} = \frac{1}{2\pi \times 10^3 \times 0.0047\times 10^{-6}} = 33.9\,k\Omega$$



مصطلح X_c هو مقاومة في الدوائر اللي فيها (C)

$$X_{CT} = X_{C1} + X_{C2}$$

كأنها مقاومة على التوالي



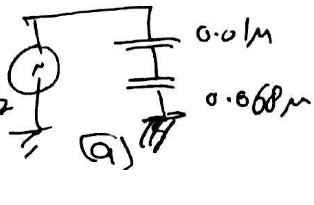
$$X_{CT} = \frac{X_{C1}\cdot X_{C2}}{X_{C1}+X_{C2}}$$

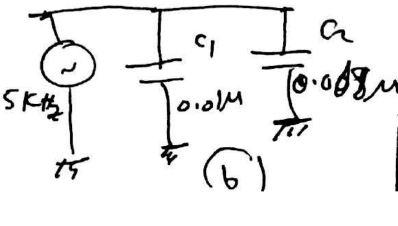
كأنها مقاومة توازي

EX(7) Calc. total capacitive reactance for circuits

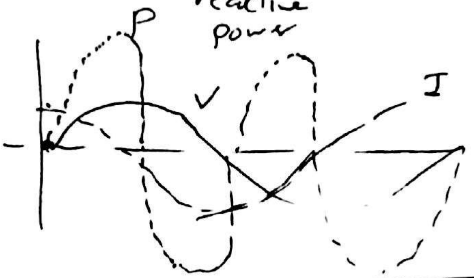
(a) $X_{CT} = X_{C1}+X_{C2} = \frac{1}{2\pi\times 5K\times 0.01\mu} + \frac{1}{2\pi\times 5K\times 0.068\mu} = 3.65K$

(b) $X_{CT} = \frac{X_{C1}X_{C2}}{X_{C1}+X_{C2}} = 408\,\Omega$





Power in capacitor $P_{reactive\ power} = V_{rms} I_{rms} = \frac{V_{rms}^2}{X_C} = I_{rms}^2 X_C$



→ True power = 0 (ideal capacitor)

صفر

Note:- يستخدم الـ capacitor في دوائر الـ filters في الـ Power supply وغيرها

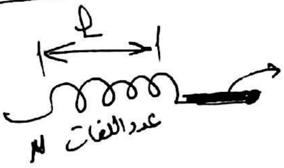
2- Inductors (Coils)

coil with (N) turns

- The induced voltage ($V_{ind} = L \frac{di}{dt}$)

inductance (henry) → L ; current rate (Amp/sec) → $\frac{di}{dt}$

- $L = \frac{N^2 \mu A}{\ell}$



صفر → $A = \pi r^2$ نصف قطر اللفة

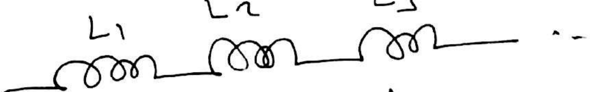
Ex

$\ell = 1.5$ cm, $\varnothing$ 0.5 cm, $\mu = 0.25 \times 10^{-3}$ H/m

$N = 350$ → $A = \pi r^2$, $r = \frac{1}{2}(0.5 \times 10^{-2})$

$$L = \frac{(350)^2 \times (0.25 \times 10^{-3}) (\pi (0.25 \times 10^{-2})^2)}{1.5 \times 10^{-2}} = 40 mH$$

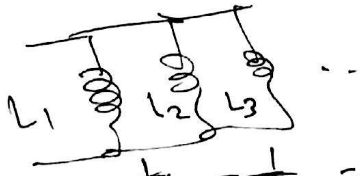
Series connection



$L_T = L_1 + L_2 + L_3 + \cdots$

تعامل معاملة المقاومات series

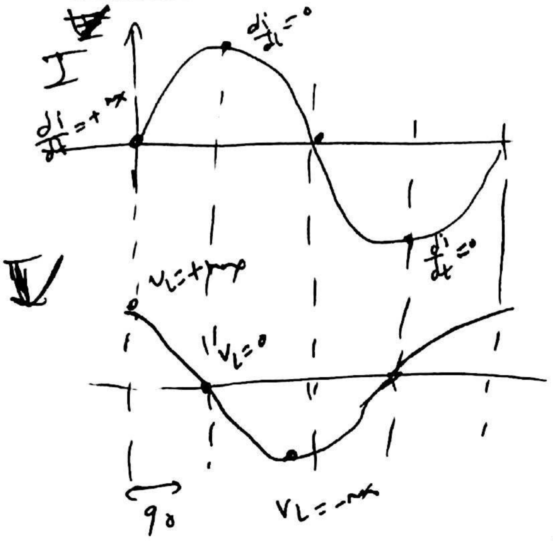
Parallel connection



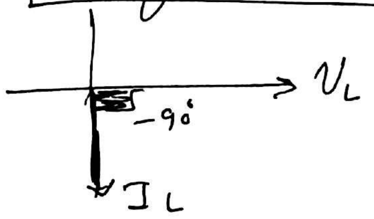
$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$

تعامل معاملة المقاومات parallel

relation between I & V in inductor



$\boxed{I \text{ lag } V \text{ by } 90^\circ}$



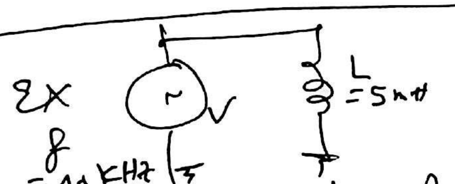
inductive reactance

$$\boxed{X_L = 2\pi f L}$$

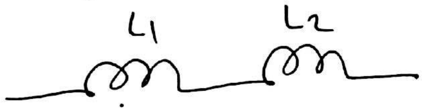
وتكتبه بالصورة المركبة

$$jX_L = j2\pi f L = j\omega L$$

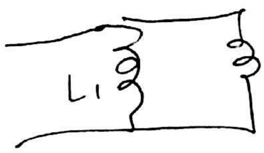
Ex



find inductive Reactance // Sol $\Rightarrow X_L = 2\pi f L = 2\pi \times 10 \times 10^3 \times (5\times10^{-3}) = 314\ \Omega$



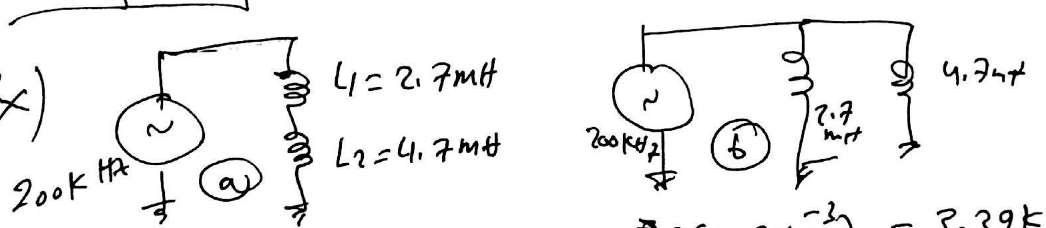
$$X_{LT} = X_{L1} + X_{L2} + \text{---}$$



$$\frac{1}{X_{LT}} = \frac{1}{X_{L1}} + \frac{1}{X_{L2}}$$

تتعامل معاملة مقاومات عادي زيها زي R
X_C

ex)



$X_{L1} = 2\pi f L_1 = 2\pi \times (200\times10^3)(2.7\times10^{-3}) = 3.39\,k\Omega$

$X_{L2} = 2\pi f L_2 = 2\pi(200\times10^3)(4.7\times10^{-3}) = 5.91\,k\Omega$

fig (a) $X_{L\,tot} = X_{L1} + X_{L2} = 9.3\,k$

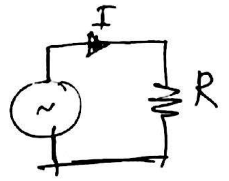
fig (b) $X_{L\,tot} = \dfrac{X_{L1}\cdot X_{L2}}{X_{L1}+X_{L2}} = 2.15\,k\Omega$

Power in inductor $P_{react} = V_{rms} I_{rms} = V_{rms}^2 / X_L = I_{rms}^2 X_L$

6 R, L, C in Circuits

1. pure Resistance
2. Pure Capacitance
3. Pure inductance
4. series RL
5. Series RC
6. series RLC
7. general case
8. Problems
9. Admittance

1 pure resistance

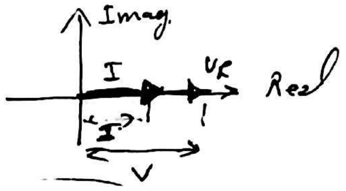


impedance

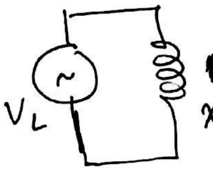
$$Z = \frac{V_R}{I} = \frac{V_R \angle 0}{I_R \angle 0} = R \angle 0$$

magnitude → phase

note: V, I in the same phase in pure Resistance

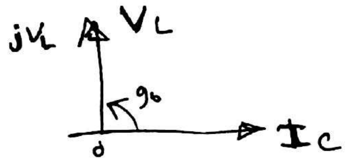


2 pure inductance



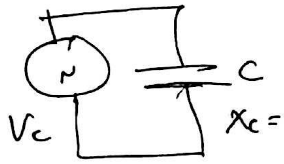
$X_L = 2\pi f L$

$$Z = \frac{V_L \angle 90}{I_L \angle 0} = X_L \angle 90 = jX_L = j\omega L = j2\pi f L$$



I lag V

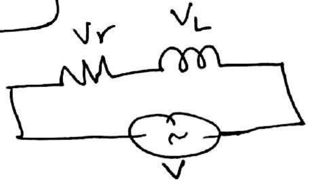
3 Pure capacitance



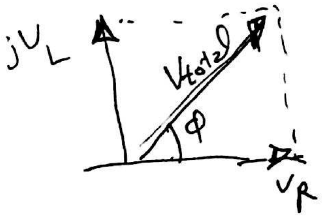
$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$Z = \frac{V_c \angle 0}{I_c \angle 90} = X_c \angle -90 = -jX_c = \frac{-j}{\omega C} = \frac{-j}{2\pi f C} = \frac{1}{j\omega C}$$

4 series RL



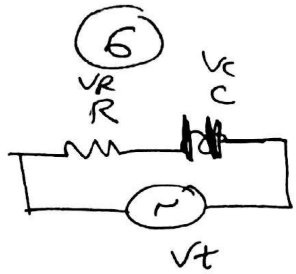
The phasor is sum of Vr & V_L
Then I lag V by angle between (0, 90°)

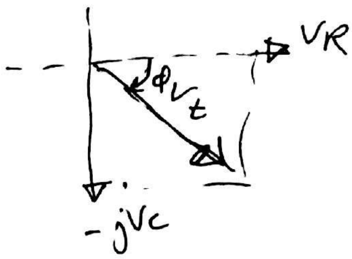


$$Z = 3 + j4 = 5 \angle \tan^{-1}\frac{4}{3}$$

3 → R; j4 → $X_L = 2\pi f L$; 5 → magnitude V_{total}; $\tan^{-1}\frac{4}{3}$ → phase (φ)

5- series RC





$V_t = V_R - jV_c$

I lead V by angle between (0, 90°)

$$\boxed{Z = R - jX_c = R - j\left(\frac{1}{\omega c}\right)}$$

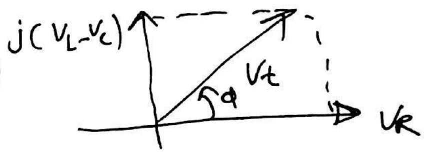
$$= \left[R + X_c = R + \frac{1}{j\omega c} = R - j\left(\frac{1}{\omega c}\right)\right]$$

magnitude $= \sqrt{R^2 + X_c^2}$, phase $\tan^{-1}\left(-\frac{X_c}{R}\right)$

6 - Series RLC

$V = V_R + j(V_L - V_c)$

$$\boxed{Z = R + j(X_L - X_c)} = \sqrt{(R)^2 + (X_L - X_c)^2} \;\Big|\; \tan^{-1}\left(\frac{X_L - X_c}{R}\right)$$



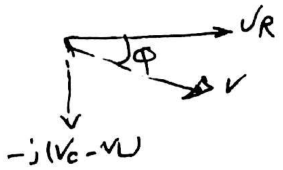
note This Circuit used in (Resonance Circuit)

There are 3 cases

1- if $X_L = X_c$ $(V_L = V_c)$ ∴ $Z = R$ (pure Resistance)
وتسمى الدائرة في تلك حالة الرنين resonance (دوائر الرنين)

2- if $X_L > X_c$ $(V_L > V_c)$ ∴ $Z = R \underbrace{+ j(X_L - X_c)}_{+ve\ value}$
circuit is inductive effect

3- if $X_L < X_c$ $(V_L < V_c)$ ∴ $Z = R + j\underbrace{(X_L - X_c)}_{-ve\ value}$
circuit is capacitive effect



Parallel AC ckt

Admittance $Y = \frac{1}{Z}$ (Siemen, S)

$Y = G \pm jB$ → susceptance (imaginary part of admittance)

↳ G → Conductance (real of admittance)

1- For Pure Resistance $\underline{Z = R}$ or $Y = \frac{1}{Z} = \frac{1}{R} = G$

2- For pure inductance $Z = jX_L$ & $Y = \frac{1}{jX_L} = \frac{-j}{X_L} = -jB_L$ → inductive susceptance

3- For pure capacitance $Z = -jX_c \Rightarrow Y = \frac{1}{Z} = \frac{1}{-jX_c} = \frac{j}{X_c} = +jB_c$

4- Series RL $Z = R + jX_L \Rightarrow Y = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$

$= \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2}$ → $B \to -\frac{X_L}{Z^2}$

↳ $G = \frac{R}{Z^2}$

5- Series RC $Z = R - jX_c$ & $Y = \frac{1}{R - jX_c} = \frac{R - jX_c}{R^2 + X_c^2}$

$= \frac{R}{R^2 + X_c^2} - j\frac{X_c}{R^2 + X_c^2}$ → $B = \frac{-X_c}{Z^2}$

↳ $G = \frac{R}{Z^2}$

6- RL parallel $\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L}$

بعد كده هيحصل إنقلابات والاختصارات هيطلع ال

$Y = \frac{1}{Z} = \frac{1}{R} - \frac{j}{X_L} = \frac{1}{R} - j\frac{1}{X_c} \to B = -\frac{1}{X_c}$

$\frac{1}{R} \to G$

7- RC Parallel $\frac{1}{Z} = \frac{1}{R} + \frac{1}{-jX_c}$

بعد الاختصارات

$Y = \frac{1}{Z} = \frac{1}{R} + \frac{j}{X_c}$ → B

$\frac{1}{R} \to G$

8

Problems

(Slide 66 → 90)
≡ sheet (1)

EX (#)-

determine the values of the resistance & series connected inductance or capacitance for each of the following impedances:-

(a) $12 + j5\ \Omega$ (b) $-j40\ \Omega$ (c) $30\angle 60°$ (d) $2.2 \times 10^{6} \angle -30\ \Omega$

if the freq. of the source is 50 Hz

Sol

series R & L

(a) $12 + j5 \equiv R + jX_L$ $\therefore$ $\boxed{R = 12\ \Omega}$

$X_L = 5 = 2\pi f L = 2\pi \times 50\, L$ $\therefore$ $\boxed{L = 0.0159\ H}$

(b) $-j40$ = pure capacitance only $X_C = 40 = \frac{1}{2\pi f C}$ $\therefore C = 79.6\ \mu F$

(c) $30\angle 60 = 30[\cos 60 + j\sin 60] = 15 + j25.98$

= series R, L $\Rightarrow$ $\boxed{R = 15\ \Omega}$ $X_L = 25.98 = 2\pi f L$

$\boxed{L = 0.0827\ H}$

(d) $2.2 \times 10^{6} \angle -30 = 2.2 \times 10^{6} [\cos -30 + j\sin(-30)] = 1.905 \times 10^{6} - j1.1 \times 10^{6}$

series R & C $\therefore$ $\boxed{R = 1.905\ M\Omega}$

$X_C = 1.1 \times 10^{6} = \frac{1}{2\pi f C}$ $\therefore C = 2.89 \times 10^{-9} F$

$= 2.89\ nF$

9

EX(2) :- determine in Polar & rectangular forms the Current flowing in inductor of negligable resistance & inductance 159.2 mH when it is connected to 250 V, 50 Hz supply

Sol/

inductive

$X_L = 2\pi f L = 2\pi(50)(159.2 \times 10^{-3}) = 50\,\Omega$

$Z = R + jX_L = 0 + j50 = 50 \angle 90^\circ\ \Omega$

$V = 250 \angle 0 = 250 + j0$

$I = \dfrac{V}{Z} = \dfrac{250 \angle 0}{50 \angle 90} = 5 \angle -90^\circ\ A$

$\boxed{A + jB \quad \text{Polar form} \qquad \sqrt{A^2+B^2}\,\angle \tan^{-1} B/A \quad \text{rectangular form}}$

EX(3) A 200 V, 50 HZ supply is connected across a Coil with inductance 0.15 H & has a series resistance of 32 Ω, determine

→ impedance of the circuit
→ current & phase
→ p.d (potential difference) across 32 Ω
→ " " " coil

Sol

(a) $X_L = 2\pi f L = 2\pi(50)(0.15) = 47.1\,\Omega$

$Z = R + jX_L = 32 + j47.1 = 57 \angle 55.81$

(b) $I = \dfrac{V}{Z} = \dfrac{200 \angle 0}{57 \angle 55.81} = 3.51 \angle -55.81^\circ$

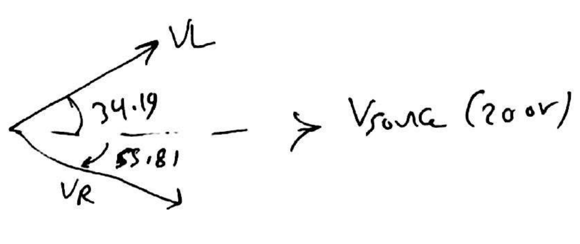
(c) P.d in 32 Ω $V_R = IR = 3.51 \angle -55.81 \times (R)$

لأن I هو نفسه
series

$V_R = (3.51 \angle -55.81)(32 \angle 0) = 112.3 \angle -55.81$

(d) p.d across coil $= IX_L = 3.51 \angle -55.81 \times 47.1 \angle 90 = 165.3 \angle 34.19$

V_L



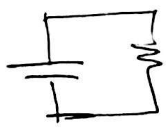
Parallel

EX(4) The admittance of a Circuit is $(0.04 + j0.025)\,S$, determine The value of $\underline{R}$ & $\underline{C}$ if they are Connected → Parallel / → series & Draw diagram if $f = 50Hz$

Sol

a-Parallel

$$Y = \underset{\hookrightarrow G}{0.04} + j\underset{\hookrightarrow B}{0.025} = \frac{1}{R} + j\left(\frac{1}{X_c}\right)$$



$$\frac{1}{R} = 0.04 \qquad \therefore R = \frac{1}{0.04} = 25\,\Omega$$

$$\frac{1}{X_c} = 0.025 \qquad \text{or } X_c = 40\,\Omega = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50\, C}$$

$$\therefore C = 79.5\,\mu F$$

b-Series

$$Y = 0.04 + j0.025 = \frac{1}{Z}$$

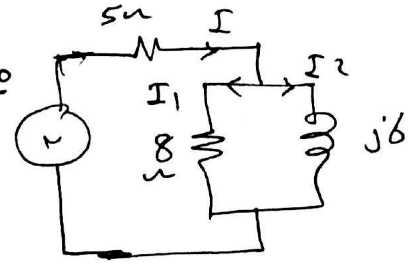
$$Z = \frac{1}{0.04 + j0.025} \times \frac{0.04 - j0.025}{0.04 - j0.025} = \frac{0.04 - j0.025}{(0.04)^2 + (0.025)^2}$$

$$= 17.98 - j11.24 = R - jX_c$$

where $R = 17.98\,\Omega$

$X_c = 11.24\,\Omega$

EX(5) Determine I, I_1, I_2 in shown circuit



$$Z_t = 5 + 8 // j6 = 5 + \frac{(8)(j6)}{8+j6}$$

$$= 5 + \frac{48j}{8+j6} = 5 + \left[\frac{48j}{8+j6} \cdot \frac{8-j6}{8-j6}\right]$$

$$= 5 + \frac{348j + 288}{[(8)^2 + (6)^2] = 100} = 5 + 3.84j + 2.88 = \underline{7.88 + 3.84j}$$

$$= 8.77 \angle 25.98^\circ$$

$$I = \frac{V}{Z} = \frac{50\angle 0}{8.77\angle 25.98} = 5.7\angle -25.98^\circ \text{ A}$$

$$I_1 = I * \frac{j6}{8+j6} \quad \text{Current divider} \quad = (5.7)\angle -25.98^\circ \times \frac{6\angle 90^\circ}{10\angle 36.87^\circ}$$

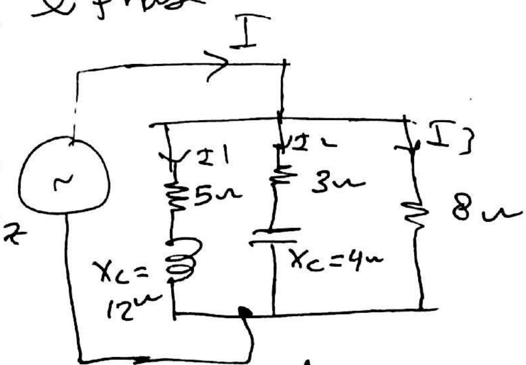
تحويل للعكوره ، نص

$$I_1 = \frac{5.7 \times 6}{10} \angle -25.98 + 90 - 36.87 = 3.42 \angle 27.15^\circ \text{ A}$$

$$I_2 = I\left(\frac{8}{8+j6}\right) = 5.7\angle -25.98^\circ * \frac{8\angle 0}{10\angle 36.87}$$

$$= 4.56 \angle -62.85^\circ \text{A}$$

EX(6) For shown circuit determine I & phase



$Z_1 = 5 + j12$ $\quad Z_2 = 3 - j4$ $\quad Z_3 = 8$

$Y_T = Y_1 + Y_2 + Y_3$ (parallel بالتعويض Y)

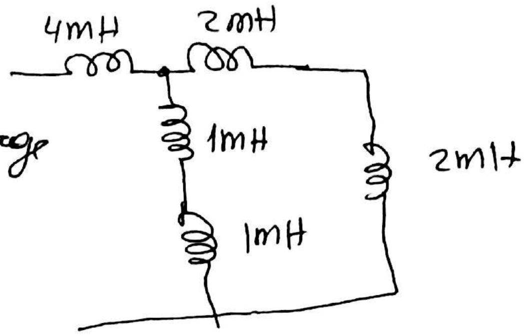
$$= \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{5+j12} + \frac{1}{3-j4} + \frac{1}{8}$$

$$Y_t = \frac{5-j12}{5^2+12^2} + \frac{3-j4}{3^2+4^2} + \frac{1}{8} = (0.2746 + j0.089)\text{ S} = 0.2887\angle 17.96^\circ$$

$$I = \frac{V}{Z} = VY = 40\angle 0^\circ \times 0.2887\angle 17.96 = 11.548\angle 17.98$$

∴ I lead supply voltage by 17.98°

نوص لي ضغط لا خضرة



Q) what frequency will produce a total current (rms) of 500 mA if RMS i/p voltage = 10V

Sol/ $Z = \frac{V}{I} = \frac{10}{500\,mA} = 20 = X_{Leq}$

$L_{eq} = (4mH) + (2mH \,//\, 4mH) = 5.33\,mH$

$X_{Leq} = 2\pi f \times 5.33\,mH = 20$

$\therefore f = \frac{20}{2\pi \times 5.33 \times 10^{-3}} = 596.8\,Hz$

Q)